Kalman Filter

CS 470 Introduction To Artificial Intelligence

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Outline

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1. Introduction
   - Background
2. Derivation
   - Properties
3. Kalman filter
   - Algorithm

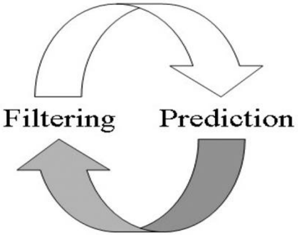
Bayesian filter

Bayesian filter family

- Gaussian noise
  - Kalman filter
  - Information filter
  - Extended Kalman filter
  - Unscented Kalman filter
- Non-Gaussian noise
  - Particle filter
  - RB Particle filter

Kalman filter

- Named by Rudolf E. Kalman
- Proposed in 1960
- Filter : filtering out the noise (uncertainty)
- Optimal estimation
- Recursive update



Outline

Assumption

System dynamics

$$
\begin{aligned}
x_k &= A_k x_{k-1} + B_k u_k + \omega_k \\
z_k &= H_k x_k + v_k
\end{aligned}
$$

- Linear model
  - transition model $x_k = A_k x_{k-1} + B_k u_k$
  - observation model $z_k = H_k x_k$
- Gaussian white noise
  - transition noise $\omega_k \sim N(0, R_k)$
  - observation noise $v_k \sim N(0, Q_k)$

What is behind?

- **transition model**

$$P(x_k \mid x_{k-1}, u_k) = det(2\pi R_k)^{-\frac{1}{2}}$$
$$exp\{-\frac{1}{2}(x_k - A_k x_{k-1} - B_k u_k)^T R_k^{-1}(x_k - A_k x_{k-1} - B_k u_k)\}$$

- **observation model**

$$P(z_k \mid x_k) = det(2\pi Q_k)^{-\frac{1}{2}} exp\{-\frac{1}{2}(z_k - Hx_k)^T Q_k^{-1}(z_k - H_k x_k)\}$$

- **prior**

$$P(x_0) = det(2\pi \Sigma_k)^{-\frac{1}{2}} exp\{-\frac{1}{2}(x_0 - \mu_0)^T \Sigma_k^{-1}(x_0 - \mu_0)\}$$

Bayesian filter

- Prediction

$$\hat{bel}(x_k) = \int P(x_k \mid x_{k-1}, u_k) bel(x_{k-1}) dx_{k-1}$$

- Update

$$bel(x_k) = \alpha P(z_k \mid x_k) \hat{bel}(x_k)$$

Outline

Kalman filter

- Prediction

$$\begin{aligned} \bar{\mu}_k &= A_k \mu_{k-1} + B_k u_k \\ \bar{\Sigma}_k &= A \Sigma_{k-1} A^T + R_k \end{aligned}$$

- Update

$$\begin{aligned} K_k &= \bar{\Sigma}_k H_k^T (H_k \bar{\Sigma}_k H_k^T + Q_k)^{-1} \\ \mu_k &= \bar{\mu}_k + K_k (z_k - H_k \bar{\mu}_k) \\ \Sigma_k &= (I - K_k H_k) \bar{\Sigma}_k \end{aligned}$$