MORRF*: Sampling-based multi-objective path planning

Daqing Yi Michael A. Goodrich Kevin D. Seppi

Department of Computer Science
Brigham Young University

Outline

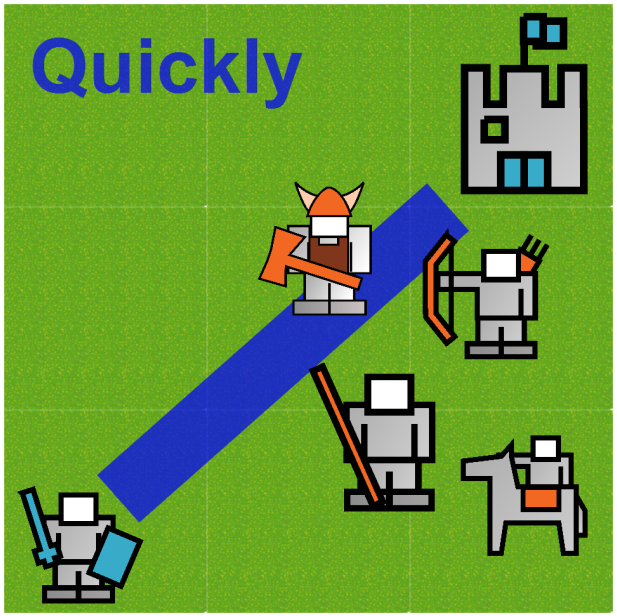
Structure

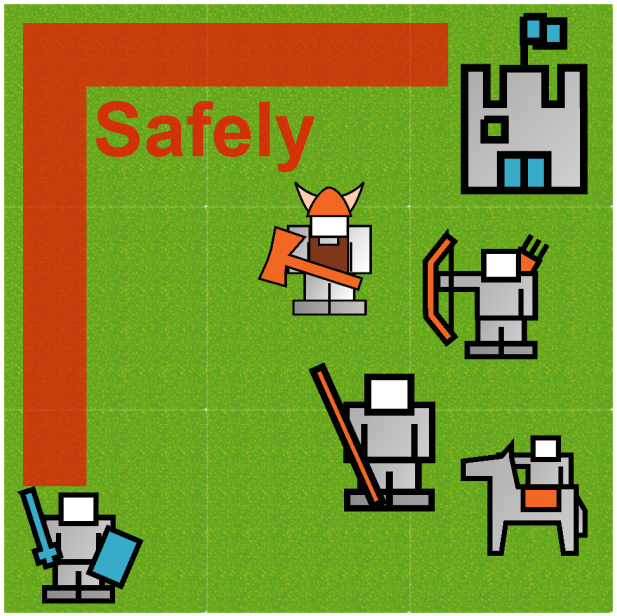
Outline

Multiple Objectives in Task

Introduction

With different objectives, there are different "best" paths.





The Need of Multi-Objective

Introduction

Multiple objectives come from a human's intent.

The Need of Multi-Objective

Introduction

Objectives

- walk quickly
- walk quietly

Constraints

- set out before it gets hot
- do not run off the path

The Difficulty in Multi-Objective

Introduction

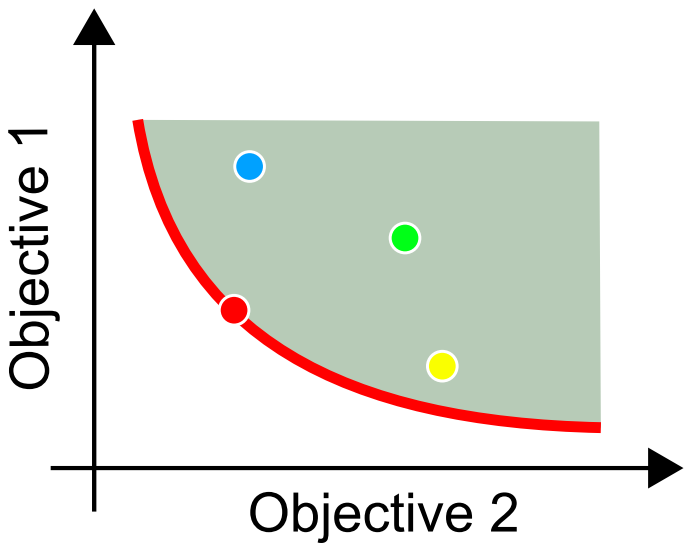
The problems in satisfying multiple objectives

- which objective is more important
- how to weigh the importance of different objectives
- how to consider indescribable preference

- Find a set of Pareto optimal set
- Interactively select the most satisfying solution

Pareto Optimal

Introduction



The lower the better

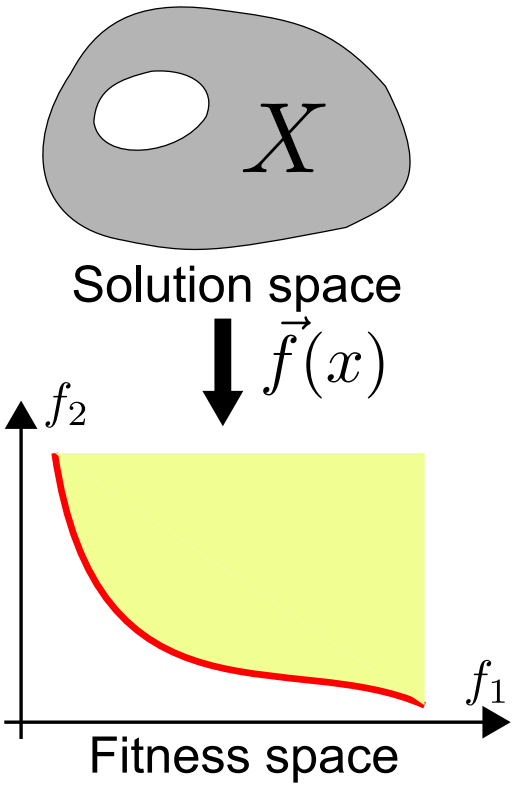
- $x_a \prec x_b$ (dominate)
  - $\forall k, f_k(x_a) \leq f_k(x_b)$
    x_a is not worse in all objectives than x_b
  - $\exists k, f_k(x_a) < f_k(x_b)$
    x_a is at least better in one objective than x_b
- non-dominant solution x^*
  - there is no other solution that can dominate x^*
    $\nexists x \in \mathbb{X}, x \prec x^*$

Finding Pareto Optimal Points is Hard

Introduction

For the point representation

- constraints
- discontinuity
- nonconvex
- optimality depends on other solutions



Methods of Multiple-Objective Optimization

Introduction

Methods

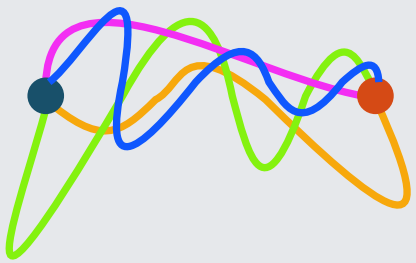
- sorting based approach : NSGA-II
- decomposition based approach : MOEA-D
- approximation based approach : PAES
- sampling based approach : ALP

Finding Pareto Optimal Paths is Harder

For the path representation

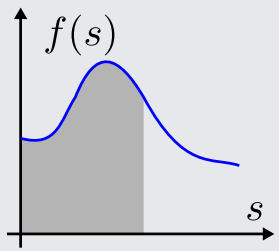
- all the properties of the point representation inherited
- format inconsistency
- fitness accumulation

Format inconsistency



It is difficult to map a path into a point in high-dimension space.

Fitness accumulation



$f(\sigma) = \int_\sigma f(s)ds$

The fitness of a path is the integral of the fitnesses of all visited points.

Outline

Graph-based Approach

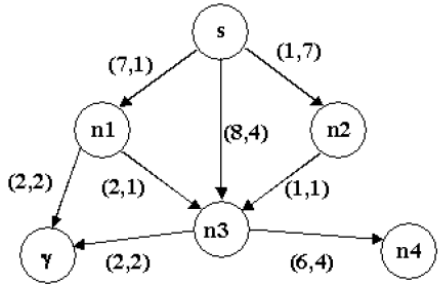
Multi-Objective Path Planning

Graph-based Approach

- Discretization → graph topology
- Vector-based cost
- Multi-objective A*

Drawbacks

- Low resolution of discretization leads to information loss
- High resolution of discretization increases the problem size



n	H(n)
s	{(3,3)}
n1	{(2,2)}
n2	{(2,2)}
n3	{(1,1)}
n4	{(0,0)}
γ	{(0,0)}

Mandow et al., "A new approach to multiobjective A* search". IJCAI'05

Point-Equivalence-based Approach

Multi-Objective Path Planning

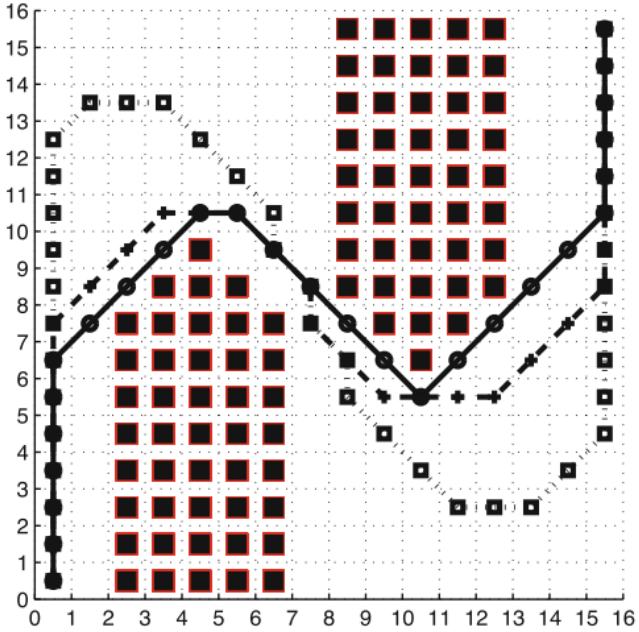
Point-Equivalence-based Approach

Model a path as a point in high dimension space

- Sequence of directions or waypoints
- Evolutionary algorithm
  - Encode into solution

Drawbacks

- obstacles → discontinuity
- giant search space



Ahmed et al., "Multi-objective path planning using spline representation". ROBIO'2011

Ahmed et al., "Multi-objective optimal path planning using elitist non-dominated sorting genetic algorithms". Soft Computing, 2013

What is an Ideal Planner

Multi-Objective Path Planning

What is an ideal planner?

- optimize in the continuous space
- return a set of Pareto optimal paths
- run in reasonable time

Our approach combines

- problem decomposition
- sampling-based optimization

Decomposition Method

MOEA-D

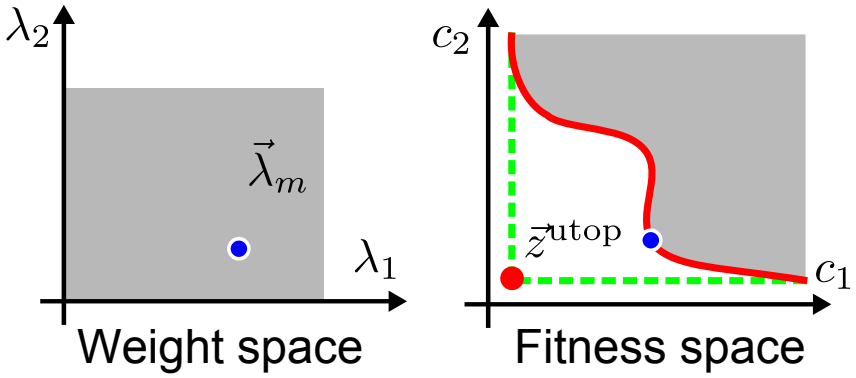
MOEA-D

Decompose a multi-objective problem into a set of single-objective subproblems

- Generating a set of different weights
- Creating one subproblem for each weight
- Solving each subproblem
- This set of solutions approximates the Pareto-optimal set

The Creation of the Subproblems

MOEA-D

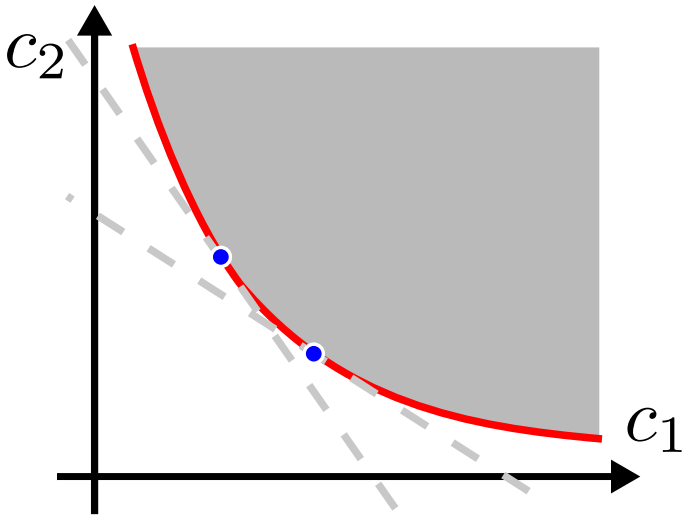


For m-th subproblem

- Sample a weight $\boldsymbol{\lambda}_m$
- Define m-th single objective function $g_m(x \mid \boldsymbol{\lambda}_m, \boldsymbol{z}^{\text{utop}})$
- Create m-th subproblem by m-th single objective function

Weighted-Sum Approach

MOEA-D



$$g_m(x) = \sum_{k=1}^{K} \lambda_k^m c_k(x)$$

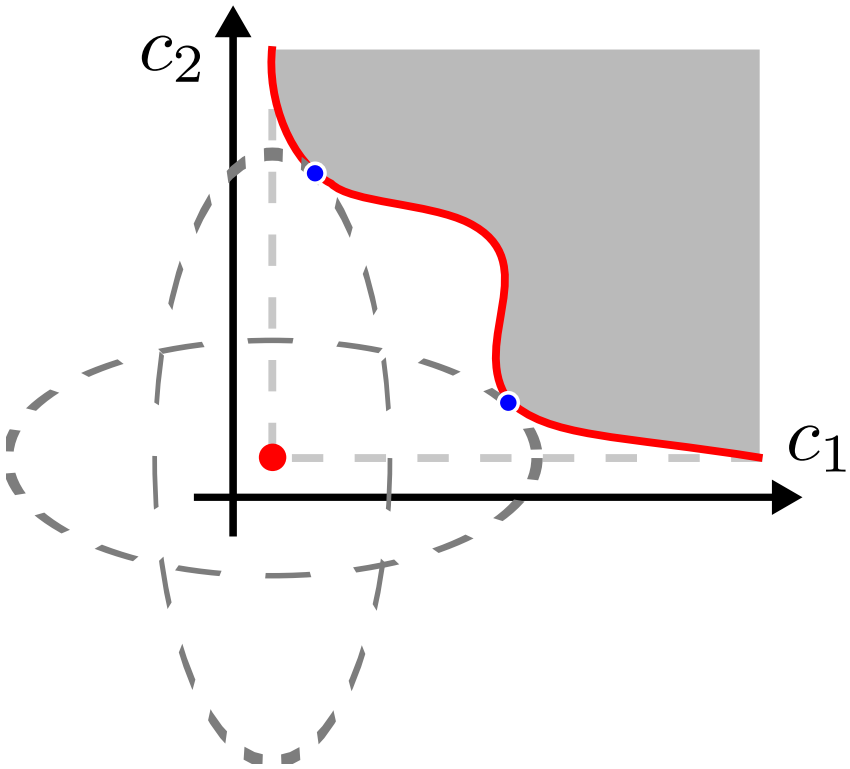
- $\arg\min_x g_m(x)$
- Require convexity

Example

- Caculate$g_m(x)$
  - $\lambda_1^m c_1(x) + \lambda_2^m c_2(x)$
- $x_m^* = \arg\max_x g_m(x)$

Tchebycheff Approach

MOEA-D



$$g_m(x) = \max_{1 \le k \le K} \{\lambda_k^m |c_k(x) - z_k^{utop}|\}$$

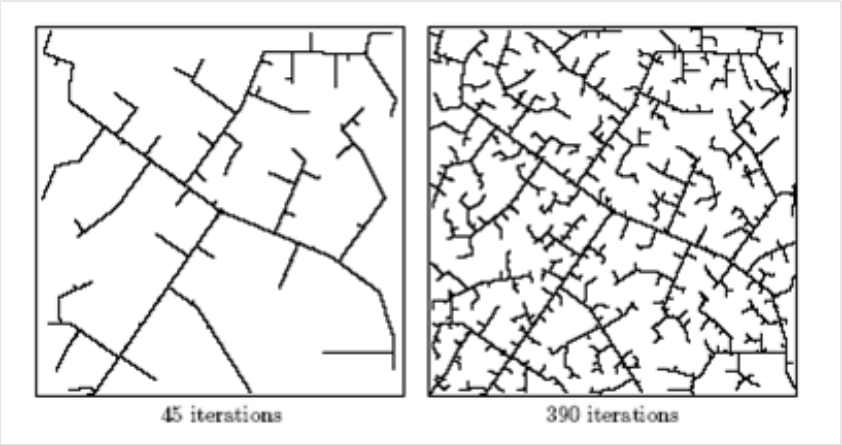
- $\arg\max_x g_m(x)$
- Support non-convexity

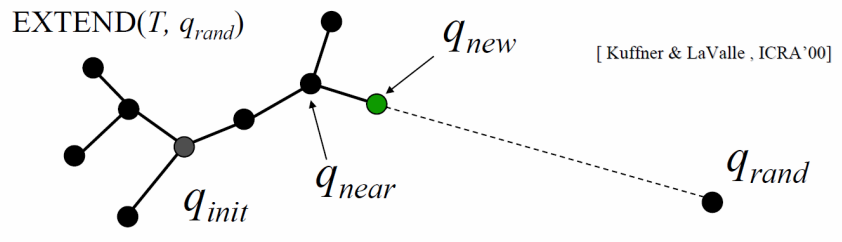
Example

- Calculate $g_m(x)$
  - $\lambda_1^m |c_1(x) - z_k^{utop}|$
  - $\lambda_2^m |c_2(x) - z_k^{utop}|$
  - take the maximum
- $x_m^* = \arg\min_x g_m(x)$

RRT
Sampling-based Path Planning

Rapidly exploring Random Tree





Process

- Sample a new vertex
- Add new vertex to the nearest vertex

Properties

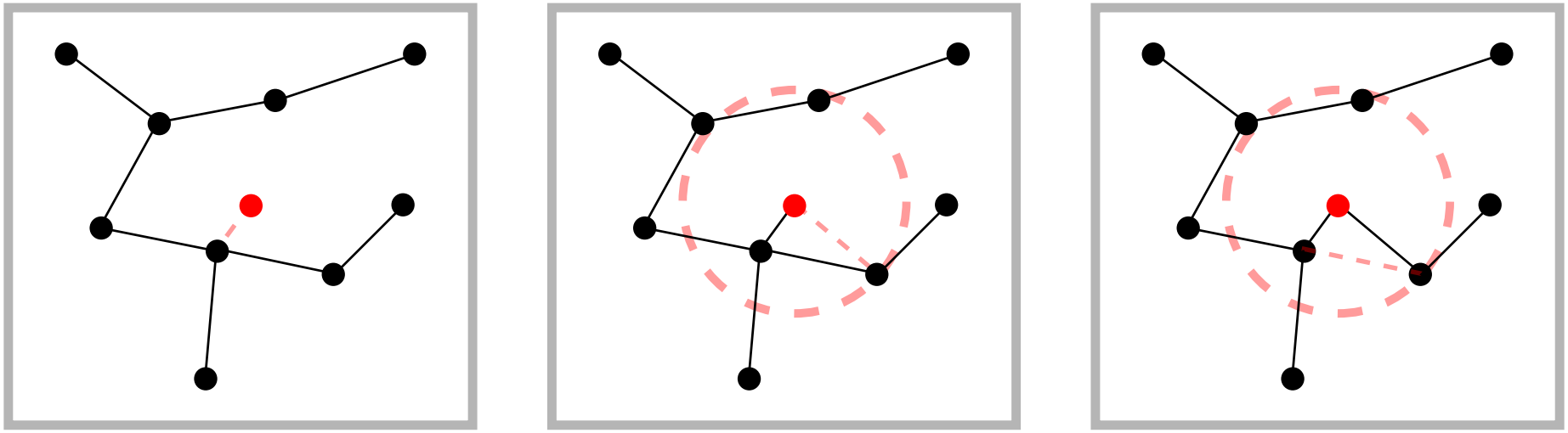
- simple (=efficient)
- high degrees of freedom
- probabilistic complete
- not asymptotically optimal

Optimal version of RRT

RRT*

Optimal Sampling-based Path Planning

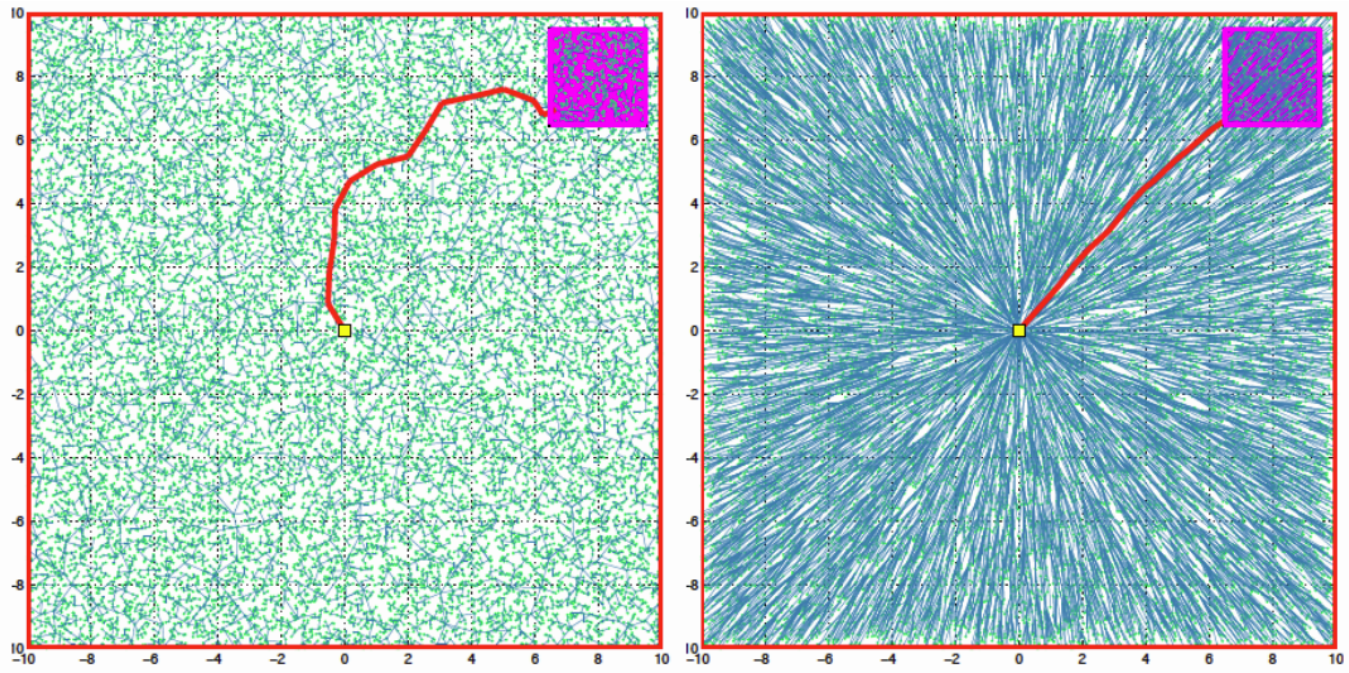
One Iteration



- Add the new vertex to its nearest vertex
- In a set of near vertices of the new vertex
  - Rewire a vertex to the new vertex if the cost-to-arrive can be reduced

RRT vs RRT*

Optimal Sampling-based Path Planning



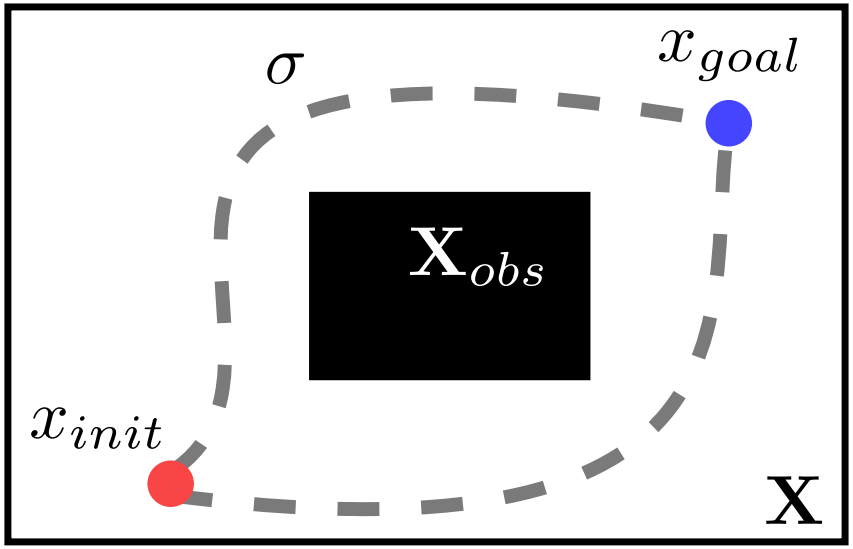
RRT **RRT***

Karaman et al., "Sampling-based algorithms for optimal motion planning". IJRR2011

Outline

Multi-Objective Path Planning

Problem



problem

- K objectives $\boldsymbol{c}(\cdot) = [c_1(\cdot), \ldots, c_K(\cdot)]^T$
- find M Pareto optimal paths $\sigma^* \in \Sigma^*$

Structure

MORRF*

Multi-Objective Rapidly exploring Random Forest*

consists of

K **reference trees** + M subproblem trees

Reference tree

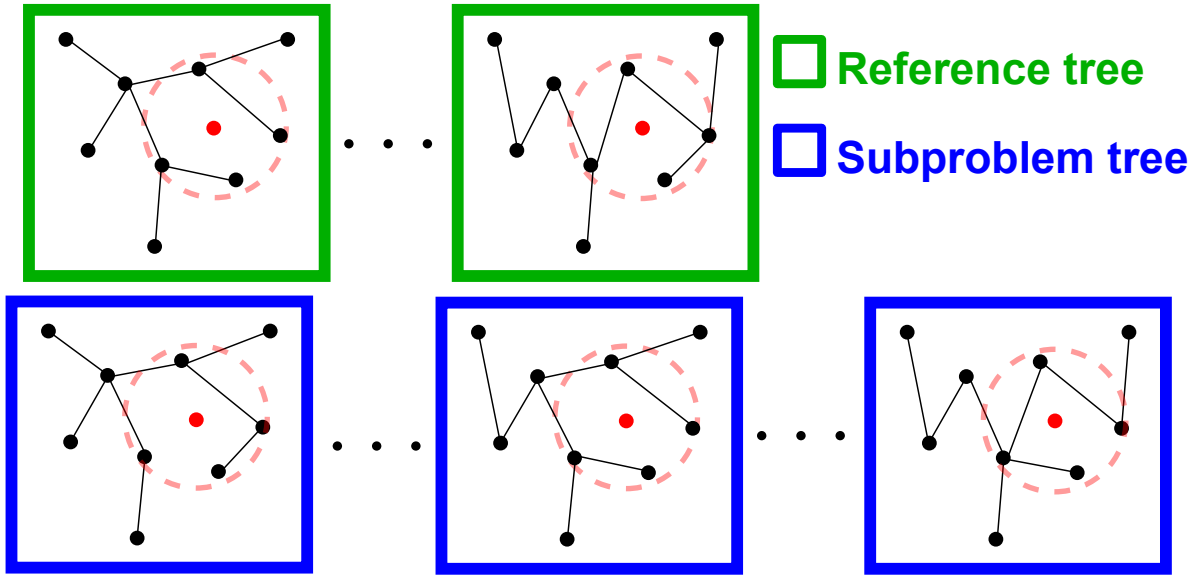
- From K objectives
- Each tree serves one objective
- Supporting the Utopia reference vector

Subproblem tree

- From M solutions
- Each tree serves one subproblem (one weight)
- This set approximates the Pareto-optimal set

Process

MORRF*



- All the trees have same vertices.
- The vertices are differently connected because of the single-objective function.

Process

MORRF*

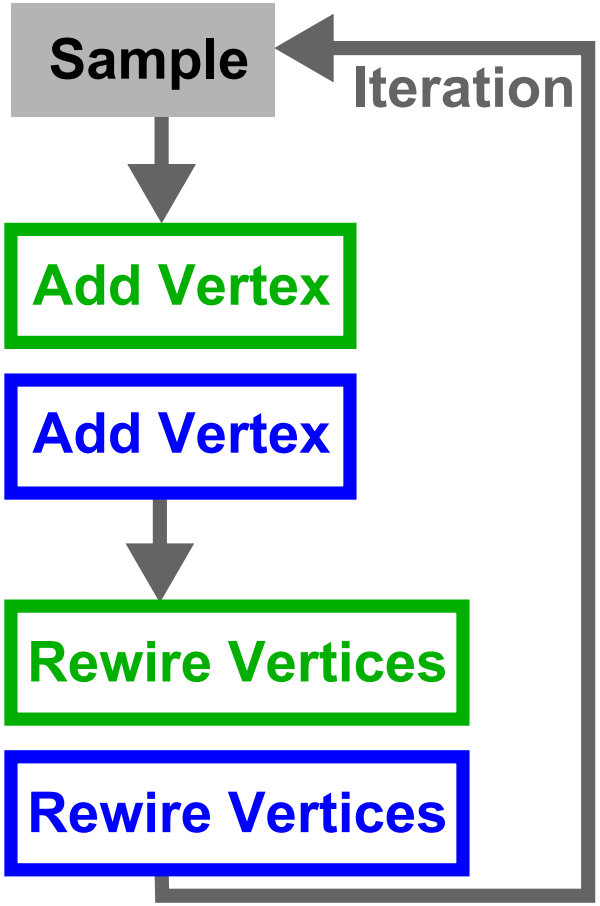
Different approaches

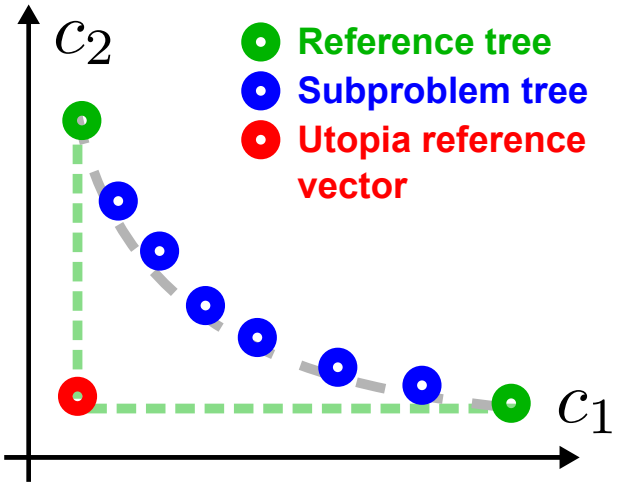
- Weighted-sum Approach
- Tchebycheff Approach

Process

MORRF*

Weighted-Sum Approach

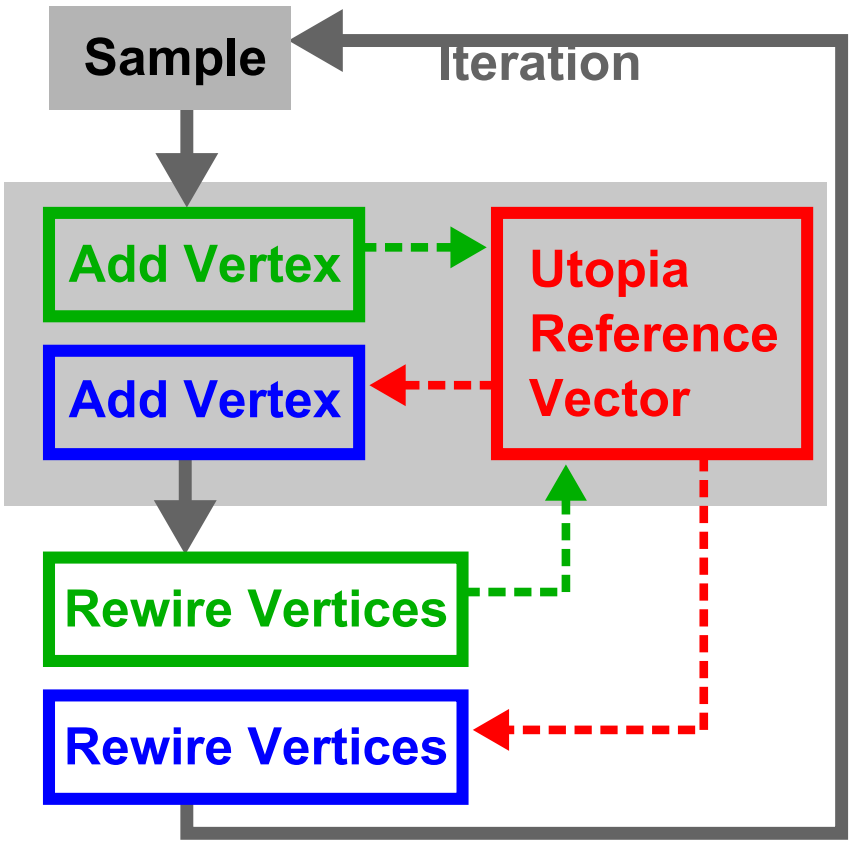


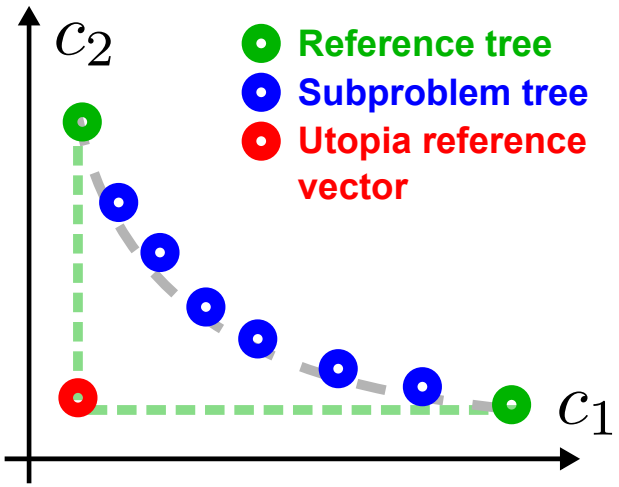


- Each reference tree returns a solution of one objective.
- Each subproblem tree returns a solution of weighted sum of all the objectives.

Process
MORRF*

Tchebycheff Approach

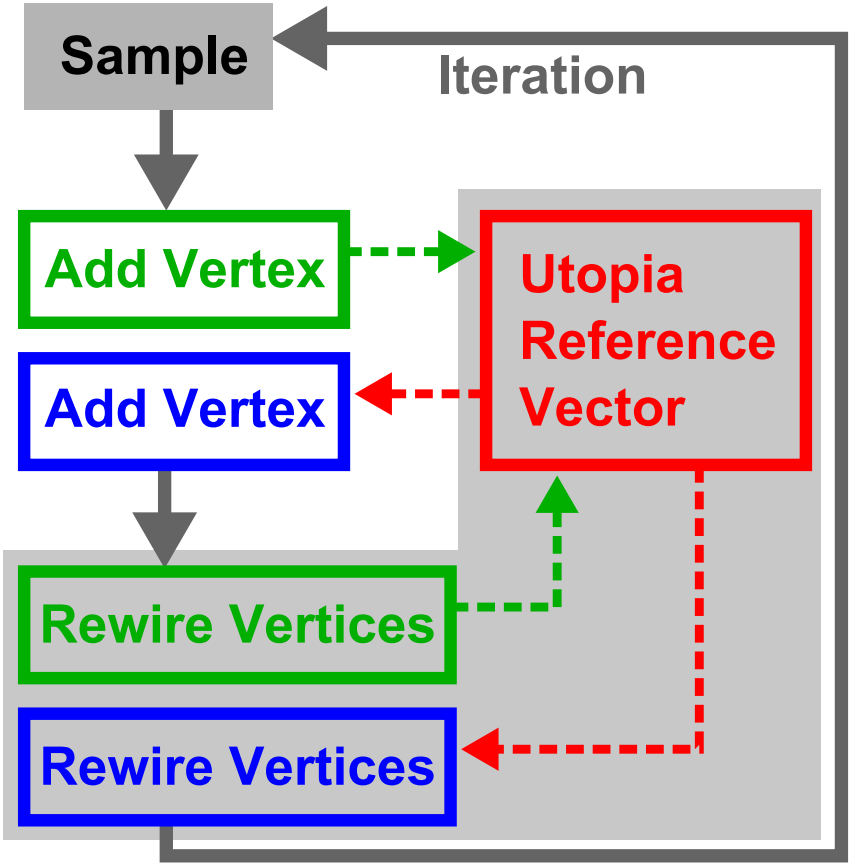


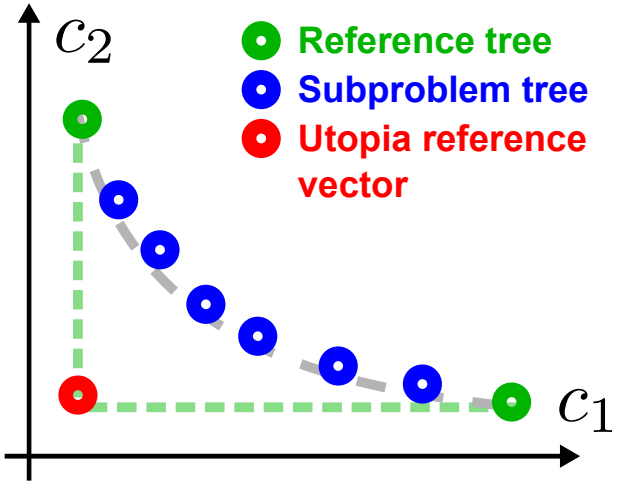


- Each reference tree adds a new vertex.
- Support the Utopia reference vector of the new position.
- According to the estimated Utopia reference vector, each subproblem tree adds a new vertex correspondingly.

Process
MORRF*

Tchebycheff Approach





- Each reference tree rewires vertices near the new vertex.
- Support the Utopia reference vector of the rewired positions.
- According to the estimated Utopia reference vector, each subproblem tree rewires vertices near the new vertex

Analysis
MORRF*

Theorem

The solutions generated by MORRF converge to a subset of the Pareto optimal set almost surely.*

Outline

How to Measure the Solutions

What are a good set of M solutions to the multi-objective path planning?

- The fitness of all the paths are on the Pareto front.
- The Pareto optimal paths show good diversity.

Compare different approaches in different cases

- run same number of iterations

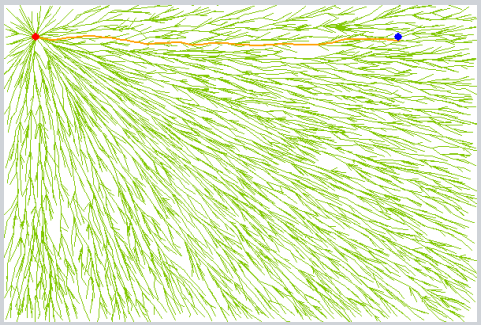
Objectives
Comparison

Compare three approaches

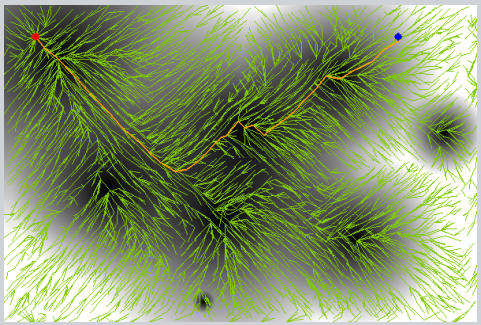
- NSGA-II
- Weighted-sum approach (MORRF*)
- Tchebycheff approach (MORRF*)

Testing with two objectives

Minimize distance



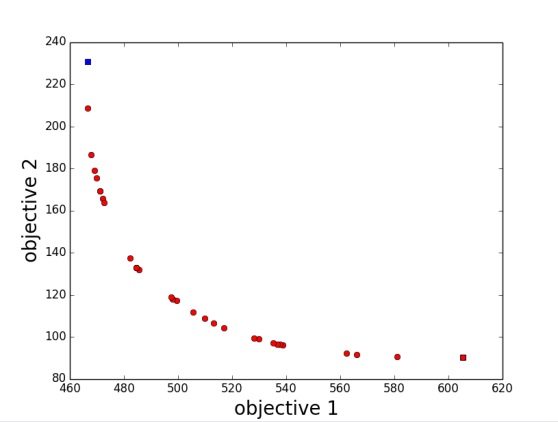
Minimize cost



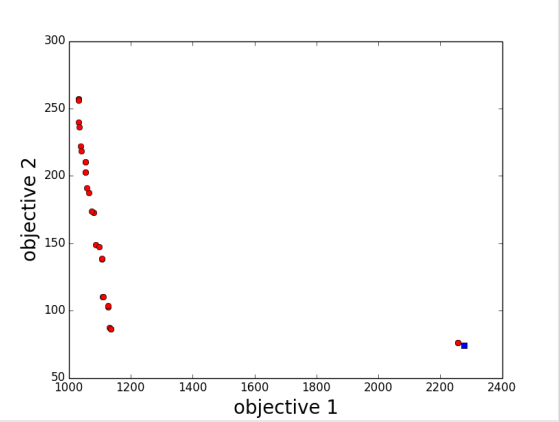
Solutions and Fitness

Comparison

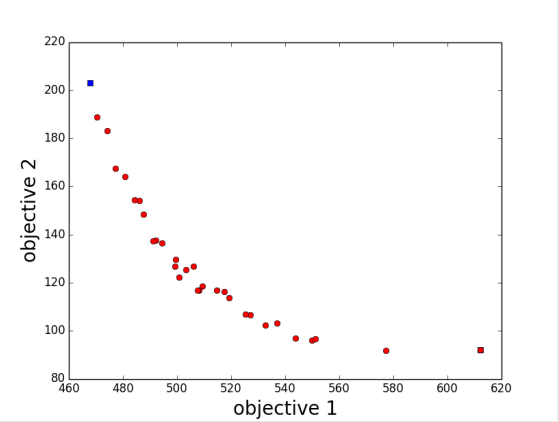
Weighted-sum approach



NSGA-II



Tchebycheff approach



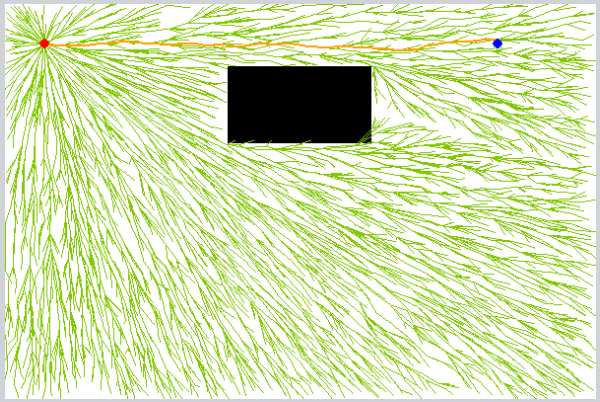
Objectives
Obstacles

Compare two approaches

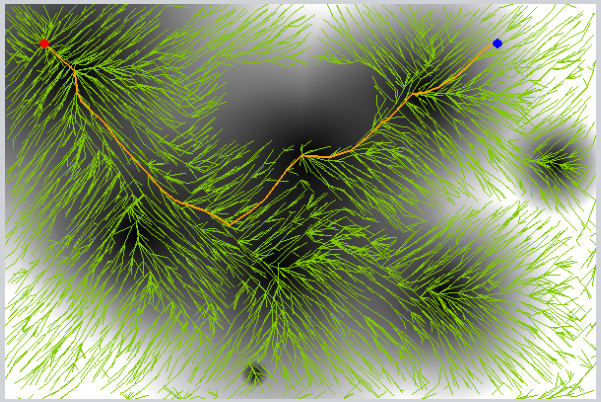
- Weighted-sum approach (MORRF*)
- Tchebycheff approach (MORRF*)

Testing with two objectives in a map with obstacle

Minimize distance



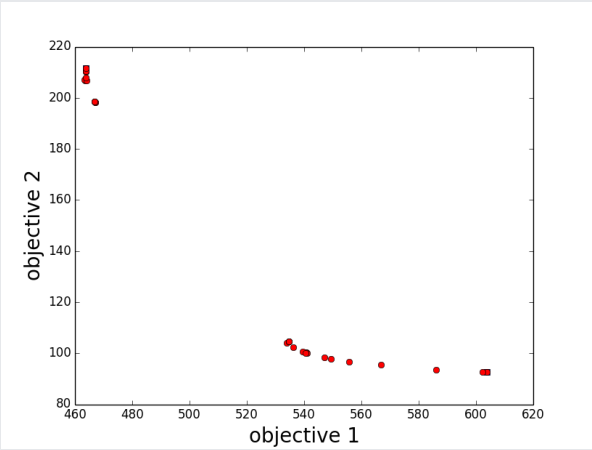
Minimize cost



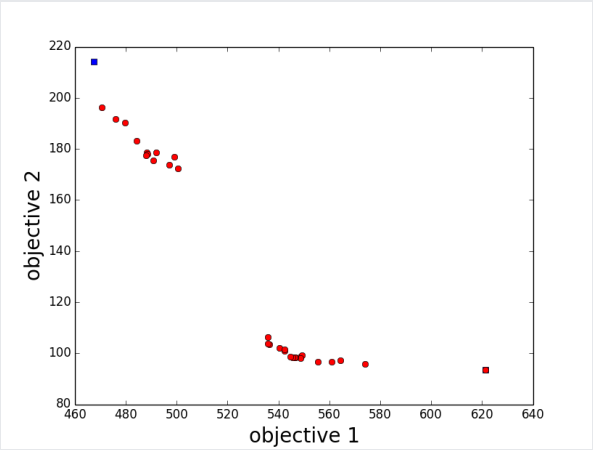
Solutions and Fitness

Obstacles

Weighted-sum approach



Tchebycheff approach



Objectives

More Objectives

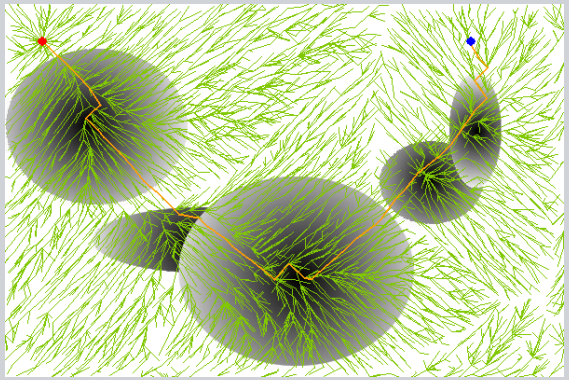
Compare two approaches

- Weighted-sum approach (MORRF*)
- Tchebycheff approach (MORRF*)

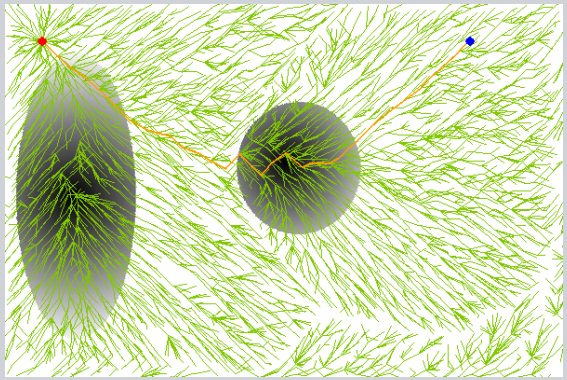
Testing with three objectives

Minimize distance

Minimize cost 1



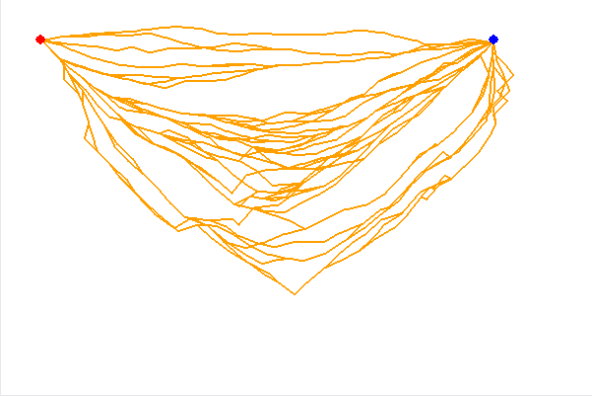
Minimize cost 2

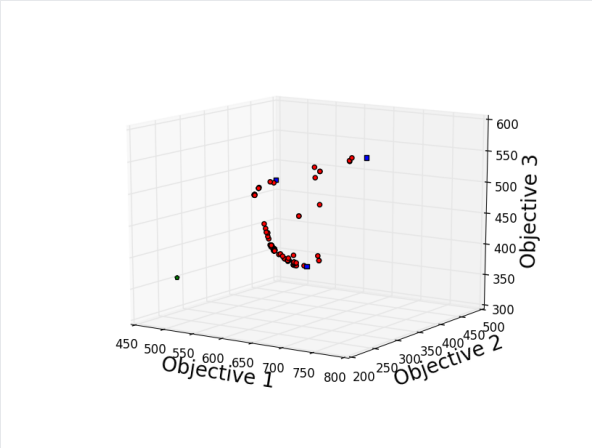


Solutions and Fitness

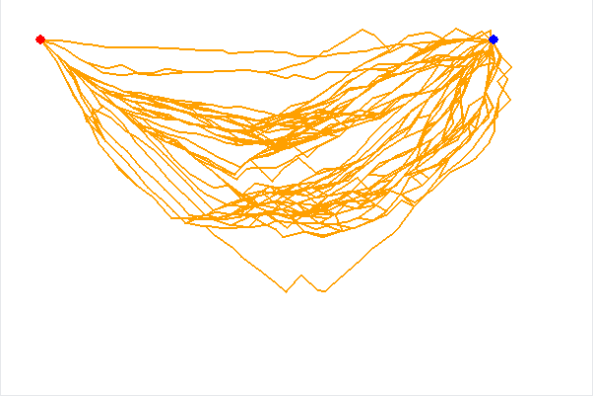
More Objectives

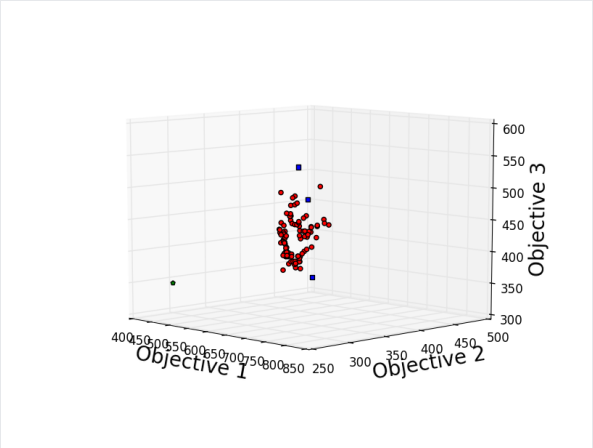
Weighted-sum approach





Tchebycheff approach





Outline

Conclusion

Features

- Approximate the set of Pareto optimal paths
- Decompose the multi-objective optimization
- Yield asymptotic optimality
- Run in reasonable time

Futurework

- Boundary intersection approach
- Informed sampling
- Multi-agent planning
- Environment uncertainty

Thank you